The Value of Accounting for Demand Seasonality in Retail Inventory Management

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Abstract

We investigate the value of accounting for demand seasonality in inventory control. We consider a single-location, single-item periodic-review lost sales inventory problem with seasonal demand in a retail environment. Customer demand has seasonality with a known season length, the lead time is shorter than the review period, and orders are placed as multiples of a fixed batch size. The cost structure comprises of a fixed cost per order, a fixed cost per batch, and a per unit variable cost to model retail handling costs. We consider four different settings which differ in the degree of demand seasonality that is incorporated in the model: with or without within-review period variations and with or without across-review periods variations. In each case, we calculate the policy which minimizes the long-run average cost and compute the optimality gaps of the policies which ignore part or all demand seasonality. We apply the problem to a real life setting, using Point-of-Sales data from a European retailer. We find that it is most beneficial to incorporate demand seasonality for high-velocity products with low batch size for which demand across review periods varies substantially and provide recommendations on how to decrease costs without significantly increasing the complexity of the retailer’s store ordering system.

Keywords: Retail inventory control, Lost sales, Non-stationary demand, Seasonality.

1 Introduction

The main challenge in managing retail inventory is to match replenishment and demand, that is providing items on the shelf justified by an upcoming shopper demand. Economies of scale in supply, inadequate store execution and demand variation often lead to out-of-stocks and excess inventory. While store execution and retail out-of-stocks have received considerable attention in academia and business practice (see Aastrup and Kotzab, 2010), the impact of demand variation has largely been overlooked (Bijvanck and Vis, 2011).

Demand in retailing is known to vary depending on the day of the week and time of year, around important holidays such as Christmas, and the seasons. For example, ice cream is in higher demand in the summer months. Demand is also generally not evenly distributed within the day.
For instance, in business districts more customers shop just after working hours on weekdays. Retailers can, to some degree, reduce demand variation, for instance by reducing promotions or offering “everyday low prices”. However, customer buying habits, like shopping on weekends, limit a retailer’s ability to completely smooth demand variation. This creates the need for retailers to account for seasonality in their inventory control and shelf inventories seasonally, in (partial) synchronization with the demand pattern (Aviv and Fedegruen, 1997).

Yet retailers lack the capabilities or skills to incorporate demand seasonality into their store ordering systems and therefore shelf replenishment. Advanced automated store ordering (ASO) systems allow for different yearly seasonality patterns for each item. However, retailers may choose to use store aggregate or chain-wide demand seasonality patterns, or to ignore demand seasonality altogether because of the added level of complexity. Less sophisticated ASO systems lack the technical ability to include seasonality effects, especially when it comes to weekly or daily seasonality. This leads to systematic mismatches in demand and supply at the item-store level, resulting in higher than needed costs. For example, Gruen and Corsten (2008) show that out-of-sync replenishment and demand lead to recurring out-of-stocks in retail stores on specific days of the week. This raises the question of the importance of incorporating demand seasonality in store ordering.

We study a setting where demand has a known seasonality pattern with two types of variations: within-review period and across-review periods (the review period is defined as the time between when two replenishment orders are placed). We apply our findings to real-life examples where demand follows a seasonality cycle over the days of the week and the times of the day. Figure 1 illustrates the seasonality patterns under investigation. It shows the sales of a specific cigarette product (global brand, single box of 10 cigarettes) at a European retail store for two months in 2010. Demand varies depending on the day of the week: weekly sales peak on Saturday and Sunday. Demand is also not evenly distributed within a day: on Saturday, most of the sales occur before 5pm while on Sunday most of the sales occurs after 5pm. In essence, the demand rate varies across days of the week and within the day, but displays the same periodic pattern every week.

Given seasonal customer demand, we research the following questions: (1) What is the cost of neglecting demand seasonality in retail inventory control? (2) For which type of products is this cost significant? (3) How much can be gained by partially incorporating demand seasonality into an ASO system and what is the best way to do so? We answer these questions in a setting which is suitable for retail environments: we consider a single location, single item, lost sales inventory problem with handling costs where the lead time is shorter than the review period, and orders are placed as multiples of a fixed batch size. Further, we assume that demand is non-stationary and seasonal. We first calculate the optimal inventory policy using the long-run average cost criterion and show how it varies with key parameters of the model. Second, we consider three alternate settings, in which the retailer accounts for varying degrees of seasonality. In the first setting, the retailer only accounts for across-review periods variations. In the second setting, the retailer only accounts for within-review periods variations. In the third setting, the retailer ignores all form
of seasonality, which often corresponds to current retail practice. We calculate the performance of the inventory policies for all three settings using the true distribution of demand and compute optimality gaps. We discuss when it is most important to incorporate within- and across-review periods variations into the ASO system. Third, we apply the problem to a real-life setting, using Point-of-Sales data from a European retailer and replenishment information. We also explore simplified policies of only distinguishing between weekday and weekend sales.

We find that it is most important to include seasonal variations for product categories with high sales velocity, large across-review period variations, and small case pack size. Ideally, a retailer should include both within- and across-review periods variations. However, if this is too difficult or costly to implement, we show that in most cases, the next best alternative is to take only across-review periods variations into account. In our real-life application, we further find that the retailer can achieve notable cost decreases by carefully defining the weekend and distinguishing between weekend and weekday demand. We also discuss some special cases where we found that ‘less information is better’, meaning that incorporating only one type of demand seasonality, e.g., accounting for within-review period variations but not across-review periods variations, can lead to worse performance than completely ignoring all levels of demand seasonality.

The closest paper to ours is a recent study by Tunc et al. (2011), who investigate the best stationary policy given that demand is non-stationary. They find that a stationary policy may be a good approximation only if demand uncertainty is high, setup costs are high and penalty costs are low. Our papers have fundamentally different approaches. Tunc et al. (2011) search for the best stationary \((s, S)\) policy given non-stationary demand: the retailer knows that demand is non-stationary but restricts himself to a stationary policy because it is easier to implement. In contrast, this paper considers the best non-stationary inventory policy given a demand pattern.

Figure 1: Sales seasonality in cigarette sales during a two-month period in 2010
that is missing some seasonality information. The retailer incorporates parts of the seasonality in its ASO, for example by using weekly demand estimates. Further, Tunc et al. (2011) use a model with backorders, no lead time, a batch size of one, no handling costs and finite horizon, which limits its applicability to the retail industry. In contrast, we consider an infinite-horizon average cost, lost-sales inventory model, with batch sizes and handling costs.

Our study contributes to the literature in several ways. First, we are, to the best of our knowledge, the first authors to consider the impact of ignoring demand seasonality in a model tailored for retailing, i.e., a model with lost sales, lead times, batch sizes, and handling costs. In our analysis we calculate the optimal policy for each possible demand setting, using dynamic programming. Second, we give insights on the tradeoff between the complexity of an ASO system and the cost benefits of incorporating demand seasonality to various degrees. Third, we include a study based on real-life sales data.

The remainder of this study is organized as follows: Section 2 gives a brief review of related single-item retail inventory models with lost sales and inventory models considering non-stationary demand. Our model is presented in Section 3. We solve the problem given the true distribution of demand in Section 4 then study the costs of ignoring non-stationarity in Section 5. Section 6 applies the model to a real-life situation. Section 7 presents the conclusions and points towards further research.

2 Literature

In their comprehensive review on lost sales inventory modeling, Bijvanck and Vis (2011) identify integrating non-stationary demand as a key concern for research. They argue that little is known about the non-stationary setting, despite the importance of understanding non-stationary demand for modeling real life phenomena. In this section, we review the literature on lost-sales inventory models and inventory models with non-stationary demand in reference to retail inventory management.

Lost-sales inventory models

In contrast to inventory models with backordering, where unmet demand is satisfied at a later time, lost-sales inventory models assume that customers are unwilling to wait for their order. Such behavior is frequent for retail out-of-stocks (Campo et al., 2000). Lost-sales models penalize unmet demand to capture the negative effects of out-of-stocks. These include diminished customer loyalty and increased in-store logistics costs through staff attending shoppers and looking for out-of-stock items in back rooms (Gruen et al., 2002).

Compared to the backordering problem, the inventory replenishment problem with lost-sales has been considered far less frequently in the literature due to its more complex structure (Bijvanck and Vis, 2011). Recent developments related to our research include incorporation of fractional
lead time. Sezen (2006) uses dynamic programming to show the long-run average cost is rather insensitive to the choice of the period length. Sezen (2006) simulates the impact of the review period length on on-hand inventory for the case of fractional lead times. He finds shorter review periods to be suitable for products with variable demand, and longer review periods suitable for products with less variable demand.

Also recently, handling costs have been added to lost-sales inventory modeling. Curscu et al. (2011) include store handling cost into the single-item inventory problem in a retail setting. This reflects Van Zelst et al. (2009)’s finding that item handling (e.g. receipt of goods, shelf replenishment) accounts for 75% of store logistics costs, while inventory accounts for the remaining 25% (see also Saghir and Jonson, 2001). Curscu et al. (2011) examine the optimal policy under the long-run average cost criterion using parameter values typical for grocery retailing. They show that retailers can achieve substantial cost reductions by including handling costs in their inventory decisions.

Non-stationary demand, however, has received very little attention in lost-sales inventory modeling. Reviewing lost-sales inventory models, Bijvanck and Vis (2011, p.11), conclude that it is necessary “to focus more research on non-stationary situations. Even though steady-state conditions are more analytically traceable, they become less relevant in practical situations due to non-stationary demand. […] Consequently, the methodology to analyze lost-sales inventory systems should change accordingly.” To fill this gap, we seek to explore a lost-sales inventory problem under non-stationary demand and review the corresponding literature.

Inventory models with non-stationary demand

Compared to models assuming stationary demand, studies on non-stationary demand inventory models are scarce (Tunc et al., 2011). Previous work on non-stationary demand settings includes Karlin (1960a,b), who treats the discounted version of the non-stationary demand problem, assuming stationary costs and variable demand. Karlin (1960b) gives an algorithm to calculate the optimal policy. Morton (1978) studies similar problems in a non-stationary setting (not necessarily periodic) and proposes that myopic and nearly myopic policies are optimal in most cases where excess stock can be discharged at low cost. Later, Zipkin (1989) considers an (uncapacitated) infinite-horizon inventory problem with stochastic demands where demand varies periodically. He introduces cycle-adjusted costs and provides a proof that a base-stock reorder policy is optimal under the average cost criterion. Aviv and Fedegruen (1997) show that in a capacitated setting, a periodically modified base-stock policy is optimal under the discounted and long-run average cost criteria. Other problem sets, such as the finite-horizon case, have also been studied (e.g. Morton and Pentico, 1995; Kauczynski and Tayur, 1998). Table 2 summarizes key approaches to modeling inventories with non-stationary demand. Note that none of these models include handling costs in their formulation.
3 The mathematical model

We consider a periodic review inventory replenishment system with lost sales, batch ordering and inventory handling costs, similar to that of Curşeu et al. (2011). The review cycle length $R$, i.e. the time between when two successive orders are placed, is exogenous to the model and set as the time unit. The lead time $L$ is fixed and shorter than the review period length. Hence, each review period is divided into two parts: a period of length $L$ before order is received, and a period of length $R - L$ after the order is received and before the next order is placed. Demands in these two parts are independent discrete random variables which exhibit a seasonal pattern with a fixed period of $\hat{d}$, that is, the demand during the lead time in review period $t$ has the same distribution as demand during the lead time in review period $t + \hat{d}$. Similarly, demand after the order is received in review period $t$ has the same distribution as demand after the order is received in review period $t + \hat{d}$. Let $D_{d,L}$ and $D_{d,R-L}$ denote the demand in the $d$-th review period within a season respectively before and after the order is received, for $d = 1, \ldots, \hat{d}$. Without loss of generality, we assume that review period 1 is the start of a new season so that the season index associated with review period $t$ is a value between 1 and $\hat{d}$ that indicates which period within the season review period $t$ corresponds to: $d_{t} = \text{mod} \ (t - 1, \hat{d}) + 1$.\(^1\) Table 2 gives the notation used in this paper.

At the beginning of the review period, the on-hand inventory $I_t$ and season index $d_t$ are observed and an order $a_t$ is placed. The order arrives $L$ time units later within the same review cycle. The order $a_t$ is a nonnegative integer multiple of a fixed batch size $q$, i.e. $a_t \in \{0, q, 2q, \ldots \}$. Next, the demand during the lead time $D_{d_t,L}$ is realized and, if possible, satisfied from on-hand inventory. All demand not immediately satisfied is lost. Then, after the order placed at the beginning of the period arrives, demand $D_{d_t,R-L}$ occurs until the beginning of the next review period. Again,

\(^1\) $\text{mod} \ (x, y)$ is the remainder of the division of $x$ by $y$. For example $d_{9} = \text{mod} \ (9 - 1, 7) + 1 = 2$. 

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Table 1: Related research on non-stationary demand inventory problems

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Review period $R$
Lead time ($0 \leq L \leq R$)
Period index, $t = 0, 1, 2, \ldots$
Season index for period $t$, $d_t = 1, 2, \ldots, \bar{d}$
Inventory on hand at the beginning of period $t = 0, 1, 2, \ldots$
Quantity ordered in period $t = 0, 1, 2, \ldots$
Demand during lead time in $d$-th review period within a season $D_{d,L}$
Demand after the receipt of the order in $d$-th review period within a season $D_{d,R-L}$
Total demand during the $d$-th review period, $D_{d,R} = D_{d,L} + D_{d,R-L}$
Fixed (exogenously determined) batch size, $q = 1, 2, \ldots$
Fixed cost per order $K$
Handling cost per batch $K_1$
Handling cost per unit $K_2$
Holding cost per unit of inventory (charged at the end of the period) $h$
Penalty cost for each unit of sales lost during a period (charged at the end of the period) $p$

Table 2: Notation

all demand not directly satisfied from on-hand inventory is lost. Figure 2 depicts the sequence of events.

The decision epoch is the beginning of each review period. The season index and the inventory on-hand at the beginning of a review period characterize the system state, so a state is $(d_t, I_t)$ and the state space is $SS = \{(1,0),(1,1),\ldots,(2,0),(2,1),\ldots, (\bar{d},0),(\bar{d},1),\ldots\}$. At each review moment, a decision is made regarding the ordering quantity limited to the set $A(i) = \{0, q, 2q, \ldots\}$, for every $i \in SS$. The expected transition times from one decision epoch to the next are deterministic and equal to $R$. The evolution of on-hand inventory from one decision epoch to the next is given by the following recursive relation:

$$I_{t+1} = ((I_t - D_{d_t,L})^+ + a_t - D_{d_t,R-L})^+, \quad d = 1, \ldots, \bar{d}; t = 0, 1, 2, \ldots,$$

where $(x)^+ = \max\{0, x\}$ for any $x \in \mathbb{R}$.

The probability $p_{d,i,j}(a)$ of a transition from state $(d, i)$ to state $(\mod(d, \bar{d}) + 1, j)$, given the
ordering decision \( a \) is:

\[
p_{d,i,j}(a) = \mathbb{P} \left( j = ((i - D_{d,L})^+ + a - D_{d,R-L})^+ \right), \quad d = 1, \ldots, \bar{d}; i, j = 0, 1, \ldots, a = 0, q, 2q, \ldots,
\]

When no order is placed in a given review period, i.e., \( a = 0 \), we have:

\[
\begin{align*}
p_{d,i,0}(0) &= \mathbb{P}(D_{d,R} \geq i), d = 1, \ldots, \bar{d}; i = 1, 2, \ldots, \\
p_{d,i,j}(0) &= \mathbb{P}(D_{d,R} = i - j), d = 1, \ldots, \bar{d}; i = 0, 1, \ldots, j = 1, 2, \ldots, i, \\
p_{0,0,0}(0) &= 1,
\end{align*}
\]

When the order amounts to \( a = nq > 0 \), we have:

\[
\begin{align*}
p_{d,i,0}(a) &= \sum_{k=0}^{i-1} \mathbb{P}(D_{d,L} = k)\mathbb{P}(D_{d,R-L} \geq i + a - k) + \mathbb{P}(D_{d,L} \geq i)\mathbb{P}(D_{d,R-L} \geq a), \\
p_{d,i,j}(a) &= \sum_{k=0}^{i-1} \mathbb{P}(D_{d,L} = k)\mathbb{P}(D_{d,R-L} = i + a - j - k) + \mathbb{P}(D_{d,L} \geq i)\mathbb{P}(D_{d,R-L} = a - j), \\
p_{d,i,j}(a) &= \sum_{k=0}^{i+a-j} \mathbb{P}(D_{d,L} = k)\mathbb{P}(D_{d,R-L} = i + a - j - k), \\
p_{0,0,0}(a) &= \mathbb{P}(D_{d,R-L} \geq a), \\
p_{d,0,j}(a) &= \mathbb{P}(D_{d,R-L} = a - j), \quad d = 1, \ldots, \bar{d}; j = 1, 2, \ldots, a, \\
p_{d,i,j}(a) &= 0, \quad \text{otherwise.}
\end{align*}
\]

The total expected cost from one decision epoch to the next (i.e. the one-period transition cost), given state \((d, i)\) and order \( a = nq \), is defined as:

\[
c_{d,i}(a) = c_{d,i}^r(a) + c_{d,i}^h(a) + c_{d,i}^p(a), \quad a \in A(i), \quad (d, i) \in SS,
\]

where the expected replenishment cost \( c_{d,i}^r \) and the expected holding \( c_{d,i}^h \) and penalty costs \( c_{d,i}^p \) are given by:

\[
\begin{align*}
c_{d,i}^r(0) &= 0, \\
c_{d,i}^r(a) &= K + K_1n + K_2nq, \quad a = nq > 0
\end{align*}
\]
and
\begin{align*}
c_{d,i}(0) + c_{d,i}^p(0) &= hE [(i - D_{d,R})^+] + pE [(i - D_{d,R})^-], \\
c_{d,i}(a) + c_{d,i}^p(a) &= hE \left([((i - D_{d,L})^+ + a - D_{d,R-L})^+] \right) \\
&+ p \left\{ E \left[ (D_{d,L} - i)^+ \right] + E \left[ (D_{d,R-L} - a - (i - D_{d,L})^+)^+ \right] \right\} \\
&= h \sum_{k=0}^{i-1} P(D_{d,L} = k)E [(i - k + a - D_{d,R-L})^+] + P(D_L \geq i)E [(a - D_{d,R-L})^+] \\
&+ p \left\{ E \left[ (D_{d,L} - i)^+ \right] + \sum_{k=0}^{i-1} P(D_{d,L} = k)E [(D_{d,R-L} - a - i + k)^+] \right\} \\
&+ pP(D_{d,L} \geq i)E [(D_{d,R-L} - a)^+],
\end{align*}

for $a > 0$, respectively, where $(x)^- = \max\{0, -x\} = -\min\{0, x\}$ for any $x \in \mathbb{R}$. The replenishment cost include a fixed component ($K$), a variable component which depend on the number of batches ($K_1$) and a variable component that varies with the number of units in the order ($K_2$). For more details on the calculation of handling costs, we refer to Curşeu et al. (2009). The holding cost captures the cost of excess inventory, such as financial cost of carriage and write-offs for waste. The penalty cost for unmet demand accounts for the negative effects on the retailer such as lost customer loyalty.

We aim to determine the inventory policy $U^*$, solving the following optimization problem:

$$\min_{U} C(U) = \frac{1}{R} \sum_{i \in SS} \pi_i c_i(a)$$

where $C(U)$ denotes the long-run expected average cost under policy $U$, and $(\pi_i)_{i \in SS}$ represents the steady-state distribution of the inventory on hand, provided it exists.

$C(U)$ is generally a complex function of $U$ and it may not be possible to determine the steady-state distribution in closed form. For the conditions under which an optimal policy exists under the long-run average-cost criterion, see Puterman (2005). For instance. If an optimal policy exists, then there exist relative values $(v_i)_{i \in SS}$ and the long-run average cost $g$, to satisfy the average-cost optimality equation:

$$v_i = \min_{a \in A(i)} \{c_i(a) - g + \sum_{j \in SS} p_{ij}(a)v_j\}, \quad i \in SS. \quad (5)$$

The unique relative values $(v_i)_{i \in SS}$ are obtained by providing an intimal condition, such as $v_0 = 0$. As is common in this type of inventory problem, we use a value iteration algorithm to solve the optimality equations and to determine the optimal policy (Puterman, 2005). In our numerical computations, the state space is truncated to a size sufficiently large to ensure a global optimum. The state space size is obtained by testing larger and larger sizes until the results are insensitive to the increments.
4 The WA setting

For notational convenience, we assume from now on that $D_{d,L}$ and $D_{d,R-L}$ have a Poisson distribution with mean $\lambda_{d,L}$ and $\lambda_{d,R-L}$ respectively, where $d$ is the with season index. Our model can also be implemented with any discrete distribution. Let $\lambda_{WA}$ be the vector that summarizes the demand distribution parameters used by the retailer when calculating its inventory policy:

$$\lambda_{WA} = \begin{pmatrix} \lambda_{1,L} & \lambda_{2,L} & \ldots & \lambda_{d,L} \\ \lambda_{1,R-L} & \lambda_{2,R-L} & \ldots & \lambda_{d,R-L} \end{pmatrix}$$

(6)

where the first row contains the mean values of demand during the lead time for the $\bar{d}$ periods in a season and the second row contains the mean values of demand after the order is received for the $\bar{d}$ periods in a season.

If the retailer uses vector $\lambda_{WA}$ to compute the transition probabilities $p_{d,i,j}(a)$ and cost vector $c_{d,i}(a)$, he is taking into account both the within-review-period variations and the across-review-periods variations. We refer to this setting as the WA setting, where $W$ stands for Within and $A$ for Across. In practice, this means that the retailer creates a separate bookkeeping of demand for each period within the season before and after the order is received; hence the retailer uses $2\bar{d}$ input variables in its ASO system. Let $U_{WA}$ denote the corresponding optimal policy.

4.1 Structure of the WA policy

Figure 6 depicts the structure of the optimal policy $WA$ as a function of the on-hand inventory for each period within a season (as well as three other policies which will be defined in Section 5). We used $K = K_1 = K_2 = 0$, $h = 1$, $p = 50$, $q = 1$, $\bar{d} = 7$ and the following $\lambda_{WA}$:

$$\lambda_{WA} = \begin{pmatrix} 6.79 & 1.21 & 2.14 & 3.07 & 4.00 & 4.93 & 5.86 \\ 0.30 & 0.54 & 0.77 & 1.00 & 1.23 & 1.46 & 1.70 \end{pmatrix}$$

Note that in the $\lambda_{WA}$ vector above, demand increases across review periods within a season. In particular, more demand needs to be satisfied with the order placed in review period $d+1$ compared to $d$ for $d = \{1, \ldots, 6\}$.

From Figure 6, we see that the structure of the optimal policy differs from an $(s, S, nq)$ policy as for low levels of inventory on-hand, the advice is to order the same quantity. We also see that the order quantity seems to be monotone in the inventory on-hand. This is true in most cases but not always, as also noted by Curşeu et al. (2011), Hill and Johansen (2006), and Jansen (1998).

To characterize an inventory policy, we define the optimal reorder point $s_d = \max\{i : (i,d) \in SS$ and $a(i,d) > 0\}$ for season index $d$ below which it is always optimal to place an order and above

\[\text{This } \lambda_{WA} \text{ vector was generated using equations (7) and (8) with } \bar{x} = 5, \delta_W = 0.2 \text{ and } \delta_A = 1.162. \text{ Details in Section 4.2.}\]
which it is never optimal to do so and the maximum reachable stock level after the order is received in season index $d$: $I^{\text{max}}_d = \max\{i + a_{i,d} : (i, d) \in SS \text{ and } a_{i,d} > 0\}$. For example, in Figure 6, we have $(s_1, ..., s_7) = (13, 7, 10, 13, 16, 19, 21)$ and $(I^{\text{max}}_1, ..., I^{\text{max}}_7) = (14, 8, 11, 14, 17, 20, 22)$.

Figure 6 also shows that the order quantities have a tendency to increase with the season index. This is due to our choice of $\lambda_{WA}$ vector where most of the demand occurs at the end of the season. Also $s_d$ and $I^{\text{max}}_d$ are monotone for periods $d = 2, ..., 7$. The first period is a transition period between the high demand at the end of the season and the low demand at the start of the season; the order quantity is low because the demand to be served with the new order is low but the values of $s_1$ and $I^{\text{max}}_1$ are still high because high demand will deplete the existing inventory until the order is received.

### 4.2 Sensitivity analysis on the $WA$ policy

In this section, we study how the long-run average costs and the structure of the $WA$ policy vary with the model parameters.

For the cost parameters, we use $K = \{0, 10, 20\}, K_1 = \{0, 5, 10\}, K_2 = \{0, 1, 2\}, p = \{25, 50, 75\}$ and $h = 1$. The value of the cost parameters are thus set relative to the unit holding cost. For the case pack size, we use $q \in \{1, 3, 6, 12\}$. We vary the $\lambda_{WA}$ by setting $\lambda = 7$ and using the following formulas for $d \in \{1, ..., 7\}$:

$$
\lambda_{d,L} = (1 - \delta_W) \left[ \overline{\lambda} + \delta_A (\text{mod}(d - 2, \lambda) - 3) \right],
$$

$$
\lambda_{d,R-L} = \delta_W \left[ \overline{\lambda} + \delta_A (d - 4) \right].
$$

In these formulas $\overline{\lambda}$ is the average demand per review period, $\delta_W$ is a parameter that allows for the proportion of demand within a review period to vary in a systematic way and $\delta_A$ is a parameter that allows the demand in each period within a season to vary in a systematic way. Increasing $\delta_W$ leads to more demand after the lead time (within each review period) compared to during the lead time. Increasing $\delta_A$ leads to more demand at the end rather than the start of the season. Our choice of values for $\lambda_{d,L}$ and $\lambda_{d,R-L}$ implies that more demand needs to be satisfied with the order placed in review period $d + 1$ compared to $d$ for $d = 1, ..., 6$.\footnote{The order placed at the start of review period $d$ has to cover the $R - L$ portion of review period $d$ plus the $L$ portion of review period $d + 1$. Our choice of values for $\lambda_{d,L}$ and $\lambda_{d,R-L}$ imply that $\lambda_{d,R-L} + \lambda_{d+1,L}$ is increasing in $d$.} Note that in order for all the mean demand values to be positive, we need $\delta_A \leq \frac{\overline{\lambda}}{3}$. We use $\overline{\lambda} = \{0.1, 1, 5, 10\}$, $\delta_W \in \{0, 0.1, ..., 0.9, 1\}$ and $\delta_A \in \{0, \frac{\overline{\lambda}}{30}, \frac{2\overline{\lambda}}{30}, ..., \frac{\overline{\lambda}}{3}\}$. Figure 3 illustrates how the $\lambda_{WA}$ vector varies with $\delta_W$ and $\delta_A$. Based on these parameters, we generated 17,424 problem instances in our sensitivity analysis, resulting in 69,696 policy and average cost calculations.

We find that the optimal reorder points $s_1, ..., s_\lambda$ are increasing in $p$ and $\overline{\lambda}$ and decreasing in $K$ and $q$. Also, the maximum reachable inventory values $I^{\text{max}}_1, ..., I^{\text{max}}_7$ are increasing in $p$, $\overline{\lambda}$, $K$ and $q$.\footnote{The order placed at the start of review period $d$ has to cover the $R - L$ portion of review period $d$ plus the $L$ portion of review period $d + 1$. Our choice of values for $\lambda_{d,L}$ and $\lambda_{d,R-L}$ imply that $\lambda_{d,R-L} + \lambda_{d+1,L}$ is increasing in $d$.}
Figure 3: Construction of the $\lambda_{WA}$ vector as a function of $\lambda$, $\delta_W$ and $\delta_A$.

$q$. These results are consistent with the findings of Curșeu et al. (2011) in a model without demand seasonality. We also find that both $s_d$ and $I_d^{\text{max}}$ values are decreasing in $\delta_W$ for $d = 1, \ldots, \bar{d}$. This is because there is less demand uncertainty when a smaller proportion of the demand occurs during the lead time as opposed to after the order is received. Finally, an increase in $\delta_A$, which has the effect of switching more of the demand towards the end of the season, creates more variance in the vectors $(s_1, \ldots, s_{\bar{d}})$ and $(I_1^{\text{max}}, \ldots, I_{\bar{d}}^{\text{max}})$, with generally higher values at the end of the season.

Figure 4 shows the results of the sensitivity analysis on the long-run average cost $C(U_{WA})$. As expected, we find that the long-run average cost increases in $K$, $K_1$, $K_2$ and $p$ (keeping all other parameters constant). We also find that the average cost increases with the case pack size $q$. This is because the case pack size represents a minimal order quantity, which limits the retailer’s flexibility in terms of order quantities. We find that the long-run average cost increases with average demand $\bar{\lambda}$, but more interestingly, it decreases with $\delta_W$. This is because increasing $\delta_W$ means that less of the review period demand occurs during the lead time, leading to fewer mid-period stock-outs. Finally, we find that the impact of $\delta_A$ on the average cost is much less pronounced than that of $\delta_W$. Though it is not perfectly monotone, it seems that increasing $\delta_A$, which has the effect of switching more of the demand towards the end of the season, eventually leads to a slight increase in costs. This is due to the fact that demand becomes more asymmetric, leading to more opportunities for stock-outs at the end of the season.

5 The value of accounting for demand seasonality

As outlined in the introduction, retailers may not track down demand data as finely as in what is required to compute the $WA$ policy ($2\bar{d}$ input variables). In particular, a retailer may ignore
Figure 4: Sensitivity analysis on the long-run average cost for the WA setting
within-review-period or across-review-periods variations or both. We consider three ways in which the retailer may ignore some of the relevant seasonality information.

First, we consider the case of a company which keeps a record of the demand it observes for each period of a season but, in its bookkeeping, does not distinguish between before and after the order is received. The retailer uses only \( \bar{d} \) input variables in its ASO system: one for each period in a season. This retailer finds that demand in the \( d \)-th review period within a season has a Poisson distribution with mean \( \lambda_{d,L} + \lambda_{d,R-L} \) for \( d = 1, \ldots, \bar{d} \). He then assumes that demand is evenly distributed during the review period, that is, he finds that \( D_{d,L} \) has a Poisson distribution with mean \( \frac{L}{R} (\lambda_{d,L} + \lambda_{d,R-L}) \) and \( D_{d,R-L} \) has a Poisson distribution with mean \( \frac{R-L}{R} (\lambda_{d,L} + \lambda_{d,R-L}) \) for \( d = 1, \ldots, \bar{d} \). We refer to this case as the \( \overline{WA} \) setting since it ignores within-review period variations but still acknowledges across-review periods variations.\(^4\) Let \( \lambda_{\overline{WA}} \) be the vector that summarizes the demand distribution parameters used by the retailer when calculating its inventory policy:

\[
\lambda_{\overline{WA}} = \left( \frac{L}{R} (\lambda_{1,L} + \lambda_{1,R-L}), \frac{L}{R} (\lambda_{2,L} + \lambda_{2,R-L}), \ldots, \frac{L}{R} (\lambda_{\bar{d},L} + \lambda_{\bar{d},R-L}) \right)
\]

Second, we also examine the situation where the retailer acknowledges that the demand distribution before and after receiving the order in a review period is different, but does not recognize the seasonality across review periods. In this case, the retailer uses only two input variables in its ASO system: one for each part of the review period (before and after the order is shelved). This retailer finds that demand in the \( d \)-th review period within a season has a Poisson distribution with mean \( \frac{1}{d} \sum_{d=1}^{\bar{d}} \lambda_{d,L} \) and \( D_{d,R-L} \) has a Poisson distribution with mean \( \frac{1}{d} \sum_{d=1}^{\bar{d}} \lambda_{d,R-L} \) for \( d = 1, \ldots, \bar{d} \). We refer to this case as the \( W\overline{A} \) setting, since the retailer sees within-review-period variations but does not account for across-review-periods variations. Let \( \lambda_{W\overline{A}} \) be the vector that summarizes the demand distribution parameters used by the retailer when calculating its inventory policy:

\[
\lambda_{W\overline{A}} = \left( \frac{1}{d} \sum_{d=1}^{\bar{d}} \lambda_{d,L}, \frac{1}{d} \sum_{d=1}^{\bar{d}} \lambda_{d,L}, \ldots, \frac{1}{d} \sum_{d=1}^{\bar{d}} \lambda_{d,L} \right)
\]

Third, we consider the case of a retailer which neither accounts for within-review-period nor across-review-periods variations. In this case, the retailer uses only one input variable in its ASO system: the total demand over one review period. This retailer finds that this demand has a Poisson distribution with mean \( \frac{1}{d} \sum_{d=1}^{\bar{d}} (\lambda_{d,L} + \lambda_{d,R-L}) \). He then assumes that demand is evenly distributed within each review period, that is, he finds that \( D_{d,L} \) has a Poisson distribution with mean \( \frac{L}{R} \frac{1}{d} \sum_{d=1}^{\bar{d}} (\lambda_{d,L} + \lambda_{d,R-L}) \) and \( D_{d,R-L} \) has a Poisson distribution with mean \( \frac{R-L}{R} \frac{1}{d} \sum_{d=1}^{\bar{d}} (\lambda_{d,L} + \lambda_{d,R-L}) \). We refer to this case as the \( W\overline{A} \) setting since the retailer ignores both within-review-period and across-review-periods variations. Let \( \lambda_{W\overline{A}} \) be the vector that summarizes the demand

---

\(^4\)We use the following notation: \( \overline{W} \) when the retailer ignores within-review period variations and \( W \) when the retailer accounts for within-review period variations. Similarly \( \overline{A} \) when the retailer ignores across-review periods variations and \( A \) when the retailer accounts for across-review periods variations.
distribution parameters used by the retailer when calculating its inventory policy:

\[
\lambda_{\overline{WA}} = \left( \begin{array}{cccccccc}
\frac{L}{R} \sum_{d=1}^{\bar{d}} (\lambda_{d,L} + \lambda_{d,R-L}) & \frac{R-L}{R} \sum_{d=1}^{\bar{d}} (\lambda_{d,L} + \lambda_{d,R-L}) & \cdots & \frac{L}{R} \sum_{d=1}^{\bar{d}} (\lambda_{d,L} + \lambda_{d,R-L}) \\
\frac{R-L}{R} \sum_{d=1}^{\bar{d}} (\lambda_{d,L} + \lambda_{d,R-L}) & \frac{L}{R} \sum_{d=1}^{\bar{d}} (\lambda_{d,L} + \lambda_{d,R-L}) & \cdots & \frac{R-L}{R} \sum_{d=1}^{\bar{d}} (\lambda_{d,L} + \lambda_{d,R-L}) \\
\frac{L}{R} \sum_{d=1}^{\bar{d}} (\lambda_{d,L} + \lambda_{d,R-L}) & \frac{R-L}{R} \sum_{d=1}^{\bar{d}} (\lambda_{d,L} + \lambda_{d,R-L}) & \cdots & \frac{L}{R} \sum_{d=1}^{\bar{d}} (\lambda_{d,L} + \lambda_{d,R-L}) \\
\frac{R-L}{R} \sum_{d=1}^{\bar{d}} (\lambda_{d,L} + \lambda_{d,R-L}) & \frac{L}{R} \sum_{d=1}^{\bar{d}} (\lambda_{d,L} + \lambda_{d,R-L}) & \cdots & \frac{R-L}{R} \sum_{d=1}^{\bar{d}} (\lambda_{d,L} + \lambda_{d,R-L}) \\
\end{array} \right)
\]

The \(\overline{WA}\) setting is the same as discussed in Curşeu et al. (2011) since the authors ignore any type of demand seasonality.

Table 3 summarizes the information requirements, that is the number of input variables used by the retailer in his ASO system for each setting. Example 1 illustrates the calculation of the \(\lambda\) vectors.

<table>
<thead>
<tr>
<th>Setting</th>
<th>Number of input variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>(WA)</td>
<td>2(d)</td>
</tr>
<tr>
<td>(\overline{WA})</td>
<td>(\bar{d})</td>
</tr>
<tr>
<td>(WA)</td>
<td>2</td>
</tr>
<tr>
<td>(\overline{WA})</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3: Demand information requirements (input variables) in the four settings

**Example 1.** Let \(\bar{d} = 7\) and

\[
\lambda_{WA} = \left( \begin{array}{cccccccc}
0.4 & 0.6 & 0.8 & 1.0 & 1.2 & 1.4 & 1.6 \\
1.6 & 2.4 & 3.2 & 4.0 & 4.8 & 5.6 & 6.4 \\
\end{array} \right)
\]

If \(R = 1\) and \(L = 0.4\), we have:

\[
\lambda_{\overline{WA}} = \left( \begin{array}{cccccccc}
0.8 & 1.2 & 1.6 & 2 & 2.4 & 2.8 & 3.2 \\
1.2 & 1.8 & 2.4 & 3 & 3.6 & 4.2 & 4.8 \\
\end{array} \right)
\]

\[
\lambda_{W\overline{A}} = \left( \begin{array}{cccccccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 \\
4 & 4 & 4 & 4 & 4 & 4 & 4 \\
\end{array} \right)
\]

\[
\lambda_{\overline{W\overline{A}}} = \left( \begin{array}{cccccccc}
2 & 2 & 2 & 2 & 2 & 2 & 2 \\
3 & 3 & 3 & 3 & 3 & 3 & 3 \\
\end{array} \right)
\]

Figure 5 illustrates the demand vectors as used in the four policies \(WA, \overline{WA}, W\overline{A},\) and \(\overline{WA}\) as given in Example 1.

In the \(\overline{WA}, W\overline{A}\) and \(\overline{WA}\) settings, we solve for the optimal inventory policy given the corresponding \(\lambda\) vector using the method described in Section 3.\(^5\) Let \(U_{\overline{WA}}, U_{W\overline{A}}\) and \(U_{\overline{WA}}\) denote the inventory policy in the \(\overline{WA}, W\overline{A}\) and \(\overline{WA}\) settings respectively.

\(^5\)The dynamic program can be simplified in the \(W\overline{A}\) and \(\overline{WA}\) cases because the retailer does not account for across-review-periods variations so it suffices to use \(I_t\) as a state variable.
We calculate the long-run expected average cost of each policy using the probability transition matrix and cost vector calculated based on $\lambda_{WA}$ in (6), which embraces within- and across-periods demand variations. We calculate the performance gap of each policy with respect to the optimal long-run average cost, i.e. the cost associated with the $WA$ policy, as:

$$100 \times \frac{C(U) - C(U_{WA})}{C(U_{WA})}$$

where $C(U)$ denotes the long-run average cost of policy $U$, for $U = U_{WA}, U_{WA}, U_{WA}$ and $U_{WA}$.

### 5.1 Comparison of the policies

In this section, we compare the structure of the $U_{WA}, U_{WA}, U_{WA}$ and $U_{WA}$ inventory policies. Figure 6 shows the optimal order quantities as a function of on-hand inventory for all four settings when $K = K_1 = K_2 = 0$, $p = 50$, $\lambda = 5$, $\delta_W = 0.2$ $\delta_W = 1.162$, and $\delta = 7$, i.e., in the same setup as in Section 4.2.

From Figure 6 we see that the inventory policy in the $WA$ and $WA$ settings is the same for each period within the season while it tends to increase with the season index in the other two settings. This is because the retailer ignores across-review-periods variations in the $WA$ and $WA$ settings and therefore treats every review period as identical. This means to the retailer in the $WA$ and $WA$ orders too much at the start of the season when demand is low and too little at the end of the season when demand is high. Further, a pairwise comparison of the policies in the $WA$ and $WA$ settings and the policies in the $WA$ and $WA$ settings reveals that ignoring within
Figure 6: Example of the optimal policy in all four settings
review-period variations leads to lower-than-optimal order quantities. This is because most of the 

demand occurs during the lead time (since $\delta_W = 0.2$), therefore the retailer tends to underestimate 
demand uncertainty when it assumes that demand is evenly distributed during the review period 
as is the case in the $\overline{W}A$ and $\overline{WA}$ settings.

5.2 Performance of the three policies

In this section, we study the performance of the policies which minimize the long run average cost 
in the $WA$, $\overline{WA}$ and $\overline{WA}$ settings by varying the model parameters. To do so, we use the same 
problem instances as in Section 4.2.

Figure 7 shows how the average optimality gaps of the three policies vary with each parameter. 
In most cases, the average optimality gap of the $\overline{WA}$ policy is the largest, followed by the $W \overline{A}$ 
policy, and then by the $\overline{WA}$ policy. This means that using more input variables in the ASO system 
generally leads to better decisions and lower average cost. However, this is not always the case. 
When $\delta_A$ is low, so there is little variation across periods in a season, the optimality gap of the 
$\overline{WA}$ policy can be higher than that of the $W \overline{A}$ policy. This is because there is more to gain by 
realizing that demand is not evenly spread across the review period. Also, when $\delta_W$ is high, so 
that little demand occurs during the lead time, the retailer benefits from ignoring both types of 
variations rather than accounting for within-review-period variations only. In other words, less 
information is better! This is because the retailer who ignores both types of seasonality benefits 
from two effects running in opposite directions. Not seeing that most of the demand occurs after 
the lead time results in higher than optimal order quantities. Not seeing that most of the demand 
occurs at the end of the season leads to lower than optimal order quantities in periods with higher 
season indices. Combined, the two effects partially offset each other. In contrast, the retailer who 
only ignores across-review periods variations but accounts for within-periods variations will order 
too little at the start of the season, resulting in higher stock-out penalties.

Figure 7 shows that the optimality gaps decrease in $K$, $K_1$, $K_2$ and $q$. This can be explained 
by the fact that larger fixed costs (per order or per batch) and larger case pack sizes lead to 
larger optimal order quantities (measured by $I_{\max}^d$ for $d = 1, \ldots, \tilde{d}$) as shown in Section 4.2. As a 
result it becomes less important to observe across- and within-review periods variations because 
the inventory ordered is used to cover multiple periods of demand. Moreover, the optimality gaps 
of the three policies are increasing in the penalty cost $p$. This is because a higher penalty cost 
makes the under-stocking associated with ignoring seasonality more costly. Further, an increase in 
the average demand $\overline{\lambda}$ leads to an increase in optimality gaps since more demand to satisfy means 
more opportunity for errors when seasonality is ignored.

An increase in $\delta_A$, which has the effect of shifting more of the demand to the end of the season, 
leads to worse performance of the policies in the $W \overline{A}$ and $\overline{WA}$ settings. This is in line with 
expectations, since the retailer ignores across-review periods variations in these settings. Varying 

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Figure 7: Results of the sensitivity analysis
\( \delta_W \) away from 0.5 means that the demand within a review period is distributed less evenly between the two sub periods (before and after the order is received). As a result the WA policy, which assumes demand is evenly distributed within the review period, performs worse. The impact on the optimality gaps of the WA setting is more difficult to analyze because of the two opposite effects offsetting each other as discussed above.

6 Application to a real-life setting

In this section we evaluate the value of accounting for demand seasonality using item-store level Point of Sales data collected at a European retailer for a single store in the entire year of 2010. The store stocks about 10,000 items (i.e. stock-keeping units) and is located at a major intersection, with two competitors’ stores nearby. It is open 7 days a week from 6am to 10pm.

The retailer records sales by the second and stores this data by the hour. So it is possible to observe monthly, weekly, daily and intraday demand variation. Following Van Donselaar et al. (2010), we conducted several observations on replenishment (after shelving) and on-shelf availability (before store closing) to ensure that tasks are executed as planned and sales data are not distorted by out-of-stocks.\(^6\) We did not find frequent or systematic out-of-stocks and therefore assume historical sales data to be a reasonable representation of shopper demand. In what follows we use the terms sales and demand interchangeably.

We analyze sales data at the item-level and find that demand during the week follows specific patterns. These patterns vary among items and are distinct from store sales patterns (see also the example of cigarette sales in Section 1, Figure 1). We focus our analysis on a sample of 10 items from 10 different product categories. All of these items are included in the ASO system.\(^7\)

For each of the items, shelf replenishment is structured as follows. Every evening at 5pm, the ASO system observes on-hand inventory and calculates the order quantities. The store manager either approves or overwrites the orders and sends a confirmation to the distribution center. All orders are processed in the same regional distribution center which the retailer owns and operates. There are no direct deliveries from suppliers. The order placed every evening at 5pm arrives by truck from the distribution center the next morning. Until 10am, store staff shelves all items according to the planograms. The replenishment staff is well trained in shelf replenishment. The store has a small backroom for organizing items for shelf replenishment, e.g. to dispose of excess packaging, and to organize returns of roll cages and recycling materials. Items left over from shelf replenishment (one jar of jam, two deodorants etc.) are organized in backroom bins until the next scheduled shelf replenishment.

\(^6\)We conducted unannounced and hidden observations in order to prevent pre-observation and post-observation changes in store ordering and replenishment as Gruen and Corsten (2008) suggest.

\(^7\)The ASO system does not include fresh produce and several bakery items which we therefore excluded from the analysis.
Based on our discussion with the store managers,\(^8\) it appears there is no systematic practice of overwriting the ASO’s order quantities. In particular, we believe that the store manager does not advance orders to balance workload as discussed in Van Donselaar et al. (2010). Overwriting the ASO occurs on rare occasions when store manager expects demand to deviate from the order suggestion based on her personal experience.

The ASO system generates orders in three steps. First, the system calculates an average of the past 52 weeks of sales data, excluding promotions and special events, for example the national day. Second, it divides the resulting weekly demand estimate by seven in order to obtain the daily demand estimate. Third, the system compares the demand estimate to on-hand inventory and calculates the daily order quantities according to a formula accounting for case pack size and shelf space. It follows that the current ASO system neither accounts for intra-week sales variations nor intra-day variations. To provide examples, it ignores if an item typically sells more on Saturdays than on Mondays. Also, the ASO system ignores if an item sells more at night (after the 5pm order placement) than before lunch (after the 10am replenishment of the last order).

We witnessed that the retailer is concerned by this inadequacy and currently considers refining the ASO system. The team in charge of ASO is thinking of incorporating intra-week sales variations to a certain degree by calculating two daily demand estimates: one for weekday sales (e.g. Monday-Friday) and one for weekend sales (e.g. Saturday and Sunday). However, they are unsure about the benefits of this refinement to the ASO system and unclear about whether to treat Friday sales as weekday or weekend.

**Modeling**

As mentioned earlier, the 10 items we study can be ordered every day at 5pm and the store is open every day from 6am to 10pm. Hence, we use \(R = 16\) hours = 1 (business) day for the length of the review period. Given that the new order is on the shelves by 10am, we use \((10 - 5) + (10 - 6) = 9\) hours = 9/16 day for our estimate of the lead time \(L\). Therefore, in this section, within-review-period variations mean intra-day variations while across-review periods correspond to across-days variations. Figure 8 depicts the sequence of events observed in practice. Note that the days of the week do not fully correspond to the review periods. For example, review period \(t\) in Figure 8 includes Saturday from 5pm to 10pm and Sunday from 6am to 10am. However, for simplicity, we call each review period by the day of the week where there is the most overlap. For instance, review period \(t\) is referred to as Sunday, review period \(t + 1\) as Monday, etc. Also let day 1 in a week be Sunday, day 2 be Monday, etc.

For each item, the sales data exhibit a clear seasonality pattern with a period of one week, i.e., \(d = 7\) days. Figure 1 in Section 1 gives an example of the observed seasonality patterns. We model

\(^8\)Specifically, we talked to the store head manager, replenishment and checkout staff, the corporate heads of logistics and sales as well as the forecasting and ASO team in the retailer’s headquarters.
the demand using a Poisson distribution; Figure 11 in the Appendix is an example of a histogram for one of the items we used.

The 10 items we selected cover a broad range of sales velocity, weekly and daily sales patterns, case-packs sizes, and other attributes, including global brands, national brands, regional brands, and private labels; hedonic and utilitarian items; items for immediate consumption and home storage; as well as varying degrees of shelf life. This product selection approach is common in on-shelf availability research (see, for instance, Campo et al., 2000; Gruen and Corsten, 2008). Weeks during which there was a promotion or a special event (specifically Easter, Christmas, and the national day) were ignored in our analysis.

Table 4 gives the average sales values during the lead time \(L\), after the shelving of the new order \((R - L)\) for each day of the week as well as average weekly demand and the corresponding case pack size. We also include two measures, denoted \(V_W\) and \(V_A\) which respectively measure the extent of within-review periods variations and across-review periods variations. They are calculated as follows:

\[
V_W = \frac{1}{7} \sum_{d=1}^{7} \left( \frac{9}{16} - \frac{\lambda_{d,L}}{\lambda_{d,L} + \lambda_{d,R-L}} \right)
\]

\[
V_A = CV (\lambda_{d,L} + \lambda_{d,R-L})_{d=1,...,7}
\]

where 9/16 is the percentage of time that corresponds to the lead time within a review period and \(CV\) stands for coefficient of variation.

Since the current replenishment policy in the retailer’s ASO system neither accounts for intra-day nor across-days variations, we choose to model it using the \(\overline{WA}\) policy introduced in Section 5. Note that this is not an exact representation of the retailer’s ASO system for two main reasons: (1) the policy used by the ASO system is a simpler policy which resembles an \((s, S, nq)\) policy (2) the order quantity chosen by the ASO system is not explicitly based on a minimization of the costs involved. In terms of costs, the \(\overline{WA}\) policy is therefore a lower bound on the performance of

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9Utilitarian items are primarily instrumental and functional, whereas hedonic items provide more experiential value, fun, excitement, and pleasure (see Sloot et al., 2005).
<table>
<thead>
<tr>
<th>Item</th>
<th>L</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
<th>Avg. wk. demand</th>
<th>Case pack size</th>
<th>Vw</th>
<th>Va</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy drink</td>
<td>L</td>
<td>9.58</td>
<td>8.69</td>
<td>8.5</td>
<td>9.09</td>
<td>10.19</td>
<td>8.11</td>
<td>126.52</td>
<td>24</td>
<td>0.06</td>
<td>0.22</td>
</tr>
<tr>
<td>(2.5dl can)</td>
<td>R-L</td>
<td>16.75</td>
<td>7.36</td>
<td>6.24</td>
<td>7.13</td>
<td>7.4</td>
<td>7.25</td>
<td>11.95</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Caffeinated soda</td>
<td>L</td>
<td>6.8</td>
<td>1.83</td>
<td>2.78</td>
<td>4.36</td>
<td>4.2</td>
<td>3.27</td>
<td>4.82</td>
<td>4</td>
<td>0.08</td>
<td>0.42</td>
</tr>
<tr>
<td>(1.5l bottle)</td>
<td>R-L</td>
<td>6.78</td>
<td>1.97</td>
<td>2.89</td>
<td>3.78</td>
<td>4.4</td>
<td>4.39</td>
<td>7.46</td>
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<tr>
<td>Milk</td>
<td>L</td>
<td>2.86</td>
<td>1.59</td>
<td>3.51</td>
<td>2.95</td>
<td>3.21</td>
<td>3.33</td>
<td>3.12</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>(1l tetra pack)</td>
<td>R-L</td>
<td>3.57</td>
<td>4.19</td>
<td>4.33</td>
<td>4.41</td>
<td>3.84</td>
<td>4.26</td>
<td>6.41</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Croissants</td>
<td>L</td>
<td>1.07</td>
<td>2.51</td>
<td>2.32</td>
<td>2.44</td>
<td>2.60</td>
<td>2.82</td>
<td>1.93</td>
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<tr>
<td>Lettuce</td>
<td>L</td>
<td>3.33</td>
<td>1.80</td>
<td>1.90</td>
<td>1.97</td>
<td>1.85</td>
<td>2.09</td>
<td>2.05</td>
<td></td>
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</tr>
<tr>
<td>Cigarettes</td>
<td>L</td>
<td>1.27</td>
<td>1.50</td>
<td>1.46</td>
<td>1.62</td>
<td>1.53</td>
<td>1.21</td>
<td>3.28</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(single box of 10)</td>
<td>R-L</td>
<td>3.11</td>
<td>1.93</td>
<td>1.87</td>
<td>2.05</td>
<td>2.01</td>
<td>2.03</td>
<td>2.01</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Sausage</td>
<td>L</td>
<td>0.73</td>
<td>0.32</td>
<td>0.64</td>
<td>0.71</td>
<td>0.77</td>
<td>0.73</td>
<td>1.08</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Potato chips</td>
<td>L</td>
<td>1.71</td>
<td>0.67</td>
<td>0.56</td>
<td>0.54</td>
<td>0.59</td>
<td>0.8</td>
<td>1.02</td>
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</tr>
<tr>
<td>(300gr pack)</td>
<td>R-L</td>
<td>1.6</td>
<td>0.5</td>
<td>0.25</td>
<td>0.54</td>
<td>0.46</td>
<td>0.62</td>
<td>1.44</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eggs</td>
<td>L</td>
<td>0.34</td>
<td>0.24</td>
<td>0.37</td>
<td>0.29</td>
<td>0.42</td>
<td>0.37</td>
<td>0.34</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(pack of six)</td>
<td>R-L</td>
<td>0.57</td>
<td>0.34</td>
<td>0.33</td>
<td>0.37</td>
<td>0.29</td>
<td>0.47</td>
<td>0.56</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Orange juice</td>
<td>L</td>
<td>0.4</td>
<td>0.22</td>
<td>0.26</td>
<td>0.24</td>
<td>0.23</td>
<td>0.19</td>
<td>0.26</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4x1l tetra pack)</td>
<td>R-L</td>
<td>0.59</td>
<td>0.17</td>
<td>0.44</td>
<td>0.52</td>
<td>0.44</td>
<td>0.3</td>
<td>0.56</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Information on the 10 selected items.

the policy used by the ASO system, which implies that our results on the impact of incorporating seasonality are on the conservative side.

We compare this policy to the WA policy which takes into account both within-day and across-days variations, the W policy which incorporates across-days but not within-day variations and the W policy which considers within-day but not across-days variations. The first four rows in Table 5 show the value of the vector λ in each of the four settings for the energy drink item as an example.

Calculating the minimum long-run average cost under each setting requires an estimate of the cost parameters, which in practice is a very difficult task. For this reason, we decided to vary the cost parameters in a way similar to the approach in Sections 4.2 and 5.2. We use $p \in \{25, 50, 75\}$, $K \in \{0, 5, 10, 15, 20, 25\}$, $K_1 \in \{0, 5, 10\}$, $K_2 \in \{0, 1, 2\}$, $h = 1$, leading to 162 problem instances per item. This way, we are able to calculate the value of incorporating seasonality under a wide range of cost treatments. The value of the case pack size $q$ was set equal to the actual value observed in practice for each item (see Table 4, third column from right).

Figure 9 gives the results of our analysis. For each item, we report the minimum, average, and maximum optimality gaps for the policies WA, W and W compared to the performance of the
We see that the optimality gaps of the $\overline{WA}$ policy can be substantial. The maximum value is equal to 66.27% and is achieved for the caffeinated soda. This means that the retailer's costs can be 66.27% higher than optimal when he ignores all forms of demand seasonality. Overall the gaps are largest for the caffeinated soda, the potato chips and the energy drink and lowest for eggs, croissants and the orange juice. In order to determine what makes the optimality gaps high or low, we regressed the optimality gaps of the $\overline{WA}$ policy on the following independent variables: average weekly demand, case pack size, $V_W$ and $V_A$ as defined in Table 4. The results are shown in Table 6. For the most part, we are able to confirm the results obtained in Section 5.1 based on our small sample of 10 items: the optimality gaps are increasing in the average weekly demand, and the extent of across-days variations (as measured by $V_A$) and decreasing in the case pack size. Surprisingly we find that they are decreasing in the extent of intra-day variations (as measured by $V_W$); we believe that this is due to our small sample size of 10 items. The results also suggest that intra-day variations are less important than across-days variations as the rest of our analysis confirms.

Figure 9 shows that for all 10 items, switching from the $\overline{WA}$ to the $\overline{WA}$ policy would lead to a decrease in the average optimality gaps. This is especially notable with the energy drink (from 6.81% to 1.48%), the caffeinated soda (from 16.10% to 0.87%), the sausage (from 4.83% to 1.12%) and the potato chips (from 12.98% to 0.29%). This means that the retailer has a lot to gain from incorporating across-days variations. In fact doing so alone brings the retailer within 1.85% of optimality for all items and within 1% for seven of them. Switching from the $\overline{WA}$ policy to the $\overline{WA}$ policy does not always lead to an reduction in the average optimality gaps. In particular, the

<table>
<thead>
<tr>
<th>No. of input variables</th>
<th>$\lambda$ vectors for the energy drink</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{WA}$</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>9.58, 8.69, 8.50, 8.09, 9.28, 10.19, 8.11</td>
</tr>
<tr>
<td></td>
<td>16.75, 7.36, 6.24, 7.13, 7.40, 7.25, 11.95</td>
</tr>
<tr>
<td>$\lambda_{WSS}$</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>10.17, 10.17, 10.17, 10.17, 10.17, 10.17</td>
</tr>
<tr>
<td></td>
<td>7.91, 7.91, 7.91, 7.91, 7.91, 7.91</td>
</tr>
<tr>
<td>$\lambda_{WSS}$</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>14.81, 9.03, 8.29, 8.56, 9.38, 9.81, 11.28</td>
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<td></td>
<td>11.52, 7.92, 6.45, 6.66, 7.30, 7.63, 8.78</td>
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<tr>
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<td></td>
<td>8.92, 8.92, 8.92, 8.92, 8.92, 8.92</td>
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<tr>
<td></td>
<td>9.15, 9.15, 9.15, 9.15, 9.15, 9.15</td>
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<tr>
<td>$\lambda_{WSS}$</td>
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<td></td>
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<tr>
<td></td>
<td>7.79, 7.79, 7.79, 7.79, 7.79, 8.20</td>
</tr>
<tr>
<td>$\lambda_{WSS}$</td>
<td>2</td>
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<tr>
<td></td>
<td>9.15, 9.15, 9.15, 9.15, 9.15, 9.27</td>
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<tr>
<td>$\lambda_{WSS}$</td>
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<td></td>
<td>8.65, 8.65, 8.65, 8.65, 8.65, 9.49</td>
</tr>
<tr>
<td></td>
<td>8.91, 8.95, 8.95, 8.95, 8.95, 8.91</td>
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<tr>
<td>$\lambda_{WSS}$</td>
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<td>14.29, 7.08, 7.08, 7.08, 7.08, 14.29</td>
</tr>
<tr>
<td>$\lambda_{WSS}$</td>
<td>4</td>
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<tr>
<td></td>
<td>9.59, 8.65, 8.65, 8.65, 9.59, 9.59</td>
</tr>
<tr>
<td>$\lambda_{WSS}$</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>11.69, 7.03, 7.03, 7.03, 11.69, 11.69</td>
</tr>
</tbody>
</table>

Table 5: $\lambda$ vectors for the different settings for the energy drink product.
Figure 9: Average optimality gaps of the \(W\overline{A}\), \(W\overline{A}\) and \(W\overline{A}\) policies for the 10 items
Table 6: Regression analysis

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficients</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-0.005</td>
<td>0.019</td>
</tr>
<tr>
<td>Case pack</td>
<td>-0.006*</td>
<td>0.001</td>
</tr>
<tr>
<td>Avg. wk. demand</td>
<td>0.001**</td>
<td>0.000</td>
</tr>
<tr>
<td>V_W</td>
<td>-0.413*</td>
<td>0.106</td>
</tr>
<tr>
<td>V_A</td>
<td>0.296***</td>
<td>0.033</td>
</tr>
<tr>
<td>R²</td>
<td>0.957</td>
<td></td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.922</td>
<td></td>
</tr>
</tbody>
</table>

Statistical significance at * p ≤ 0.05, ** p ≤ 0.01, *** p ≤ 0.001.

A retailer would be worse off if it was to incorporate intra-day variations without taking into account across-days variations for the energy drink, caffeinated soda, sausage, potato chips and cigarettes. This is another example of ‘less information is better’ as discussed in Section 5.1.

Switching to the WA policy may lead to almost optimal performance as measured by the WA policy, but doing so requires using 7 input variables in the inventory replenishment system, which may be too complex for some companies or their ASO system. In what follows we therefore consider a middle-ground setting in which the retailer groups the days of the week into weekdays versus weekend days and calculates a separate average daily demand for each group. As mentioned before, the retailer we spoke to considered implementing a similar idea.

We construct six new settings. These six settings differ with respect to the definition of weekend (either Friday-Saturday, Saturday-Sunday or Friday-Saturday-Sunday) and whether or not they account for intra-day variations. In settings where the retailer does not account for intra-day variations, the retailer assumes that the demand is evenly distributed throughout the review period. In his book-keeping, he keeps track of only two values of demand: the daily weekday demand and daily weekend demand. We refer to these cases as the \( \overline{W} (FS) \), \( \overline{W} (SS) \) and \( \overline{W} (FSS) \) settings when the weekend is defined respectively as Friday-Saturday, Saturday-Sunday and Friday-Saturday-Sunday. The corresponding \( \lambda \) vectors are calculated as follows:

\[
\lambda_{W(FS)} = \begin{pmatrix}
\frac{9}{16} \lambda_d & \frac{9}{16} \lambda_d & \frac{9}{16} \lambda_d & \frac{9}{16} \lambda_d & \frac{9}{16} \lambda_d & \frac{9}{16} \lambda_d & \frac{9}{16} \lambda_d \\
\frac{7}{16} \lambda_d & \frac{7}{16} \lambda_d & \frac{7}{16} \lambda_d & \frac{7}{16} \lambda_d & \frac{7}{16} \lambda_d & \frac{7}{16} \lambda_d & \frac{7}{16} \lambda_d \\
\end{pmatrix}
\]

where \( \overline{\lambda}_d = \frac{1}{5} \sum_{d=1}^{5} (\lambda_{d,L} + \lambda_{d,R-L}) \) and \( \overline{\lambda}^e = \frac{1}{2} \sum_{d=6}^{7} (\lambda_{d,L} + \lambda_{d,R-L}) \).

\[
\lambda_{W(SS)} = \begin{pmatrix}
\frac{9}{16} \lambda_d & \frac{9}{16} \lambda_d & \frac{9}{16} \lambda_d & \frac{9}{16} \lambda_d & \frac{9}{16} \lambda_d & \frac{9}{16} \lambda_d & \frac{9}{16} \lambda_d \\
\frac{7}{16} \lambda_d & \frac{7}{16} \lambda_d & \frac{7}{16} \lambda_d & \frac{7}{16} \lambda_d & \frac{7}{16} \lambda_d & \frac{7}{16} \lambda_d & \frac{7}{16} \lambda_d \\
\end{pmatrix}
\]
where \( \bar{\lambda}_d = \frac{1}{2} \sum_{d=2}^6 (\lambda_{d,L} + \lambda_{d,R-L}) \) and \( \bar{\lambda}^e = \frac{1}{3} \sum_{d=1,7} (\lambda_{d,L} + \lambda_{d,R-L}) \).

\[
\lambda_{\text{W}(FSS)} = \begin{pmatrix}
\frac{9}{16} \bar{\lambda}_e & \frac{9}{16} \bar{\lambda}_d & \frac{9}{16} \bar{\lambda}_d & \frac{9}{16} \bar{\lambda}_d & \frac{9}{16} \bar{\lambda}_d & \frac{9}{16} \bar{\lambda}_d & \frac{9}{16} \bar{\lambda}_e & \frac{9}{16} \bar{\lambda}_e
\end{pmatrix}
\]

where \( \bar{\lambda}_d = \frac{1}{2} \sum_{d=2}^5 (\lambda_{d,L} + \lambda_{d,R-L}) \) and \( \bar{\lambda}^e = \frac{1}{3} \sum_{d=1,6,7} (\lambda_{d,L} + \lambda_{d,R-L}) \).

In settings where the retailer accounts for intra-day variations, it keeps track of four quantities: weekday demand before the leadtime, weekday demand after the leadtime, weekend demand before the leadtime and weekend demand after the leadtime. We refer to these as the \( W(FS) \), \( W(SS) \) and \( W(FSS) \) settings when the weekend is defined respectively as Friday-Saturday, Saturday-Sunday and Friday-Sunday-Saturday. The corresponding \( \lambda \) vectors are calculated as follows:

\[
\lambda_{\text{W}(FS)} = \begin{pmatrix}
\delta^e \bar{\lambda}_e & \delta^e \bar{\lambda}_d & \delta^e \bar{\lambda}^d & \delta^e \bar{\lambda}^d & \delta^e \bar{\lambda}^d & \delta^e \bar{\lambda}^d & \delta^e \bar{\lambda}_e & \delta^e \bar{\lambda}_e
\end{pmatrix}
\]

where \( \bar{\lambda}_d = \frac{1}{2} \sum_{d=1}^5 (\lambda_{d,L} + \lambda_{d,R-L}) \), \( \bar{\lambda}^e = \frac{1}{2} \sum_{d=6}^7 (\lambda_{d,L} + \lambda_{d,R-L}) \), \( \delta^d = \frac{\sum_{d=1}^5 \lambda_{d,L} + \lambda_{d,R-L}}{\sum_{d=1}^5 (\lambda_{d,L} + \lambda_{d,R-L})} \) and \( \delta^e = \frac{\sum_{d=1}^7 \lambda_{d,L} + \lambda_{d,R-L}}{\sum_{d=1}^7 (\lambda_{d,L} + \lambda_{d,R-L})} \).

\[
\lambda_{\text{W}(SS)} = \begin{pmatrix}
\delta^e \bar{\lambda}^d & \delta^e \bar{\lambda}^d & \delta^e \bar{\lambda}^d & \delta^e \bar{\lambda}^d & \delta^e \bar{\lambda}^d & \delta^e \bar{\lambda}^d & \delta^e \bar{\lambda}^e & \delta^e \bar{\lambda}^e
\end{pmatrix}
\]

where \( \bar{\lambda}_d = \frac{1}{2} \sum_{d=2}^6 (\lambda_{d,L} + \lambda_{d,R-L}) \), \( \bar{\lambda}^e = \frac{1}{2} \sum_{d=1,7} (\lambda_{d,L} + \lambda_{d,R-L}) \), \( \delta^d = \frac{\sum_{d=2}^6 \lambda_{d,L} + \lambda_{d,R-L}}{\sum_{d=2}^6 (\lambda_{d,L} + \lambda_{d,R-L})} \) and \( \delta^e = \frac{\sum_{d=1,7} \lambda_{d,L} + \lambda_{d,R-L}}{\sum_{d=1,7} (\lambda_{d,L} + \lambda_{d,R-L})} \).

\[
\lambda_{\text{W}(FSS)} = \begin{pmatrix}
\delta^e \bar{\lambda}_e & \delta^e \bar{\lambda}_d & \delta^e \bar{\lambda}^d & \delta^e \bar{\lambda}^d & \delta^e \bar{\lambda}^d & \delta^e \bar{\lambda}^d & \delta^e \bar{\lambda}_e & \delta^e \bar{\lambda}_e
\end{pmatrix}
\]

where \( \bar{\lambda}_d = \frac{1}{2} \sum_{d=1}^5 (\lambda_{d,L} + \lambda_{d,R-L}) \), \( \bar{\lambda}^e = \frac{1}{3} \sum_{d=1,6,7} (\lambda_{d,L} + \lambda_{d,R-L}) \), \( \delta^d = \frac{\sum_{d=1}^5 \lambda_{d,L} + \lambda_{d,R-L}}{\sum_{d=1}^5 (\lambda_{d,L} + \lambda_{d,R-L})} \) and \( \delta^e = \frac{\sum_{d=1,6,7} \lambda_{d,L} + \lambda_{d,R-L}}{\sum_{d=1,6,7} (\lambda_{d,L} + \lambda_{d,R-L})} \).

Rows 5 to 10 in Table 5 show the value of the vector \( \lambda \) in each of the six settings for the energy drink example.

We now repeat the optimality gap analysis with these six new settings using the same 162 problem instances outlined earlier. Figure 10 gives the results of our analysis. It shows that it is possible to achieve a significant reduction in costs by distinguishing between weekdays and weekend days. In some cases however, it is important to use the appropriate definition for the weekend. For example, the weekend should clearly be defined as Saturday-Sunday for the energy drink, caffeinated soda, croissant and potato chips items, as Friday-Saturday for sausage and as Friday-
Figure 10: Average optimality gaps of the $W_A$, $W(FS), W(SS)$, $W(FSS)$, $W(FS), W(SS)$ and $W(FSS)$ policies for the 10 items
Saturday-Sunday for lettuce. Not defining the weekend properly can lead to worse performance than the \( \overline{WA} \) policy (e.g., for the energy drink, croissants, cigarettes and orange juice). Incorporating intra-day variations generally leads to a further reduction in costs but this improvement is not as important compared to the cost decrease that comes with the weekday-weekend distinction.

In summary, if the retailer has limited resources or limited capabilities for incorporating seasonal variations into its ASO system, it should mostly focus on categories of high velocity products with large across-days variations (as measured by the coefficient of variation \( V_A \)) and small case pack size, where the savings can be the highest. The first best option is to incorporate intra-day as well as across-days variations (14 input variables) but if this proves to be too difficult, the second best option is to take only across-days variations into account (7 input variables). If this is still too complicated – and our discussions with retailers suggest this may be the case – the retailer can still achieve considerable reductions in costs by carefully defining the weekend and distinguishing between weekend and weekday demand (i.e. using only 2 input variables for the ASO system).

7 Conclusions and Future Research

We have analyzed the optimal inventory decision for a retailer when its demand in non-stationary and in particular, seasonal. By comparing the results of the optimal policy with policies accounting for seasonality to a lesser degree, we have obtained insights into the cost of ignoring demand seasonality in inventory management.

Our paper yields some important insights for retail management. By accounting for non-stationary demand in inventory management, retailers can reduce inventory holding, handling and stock-out cost substantially. Cost savings are higher for fast-moving items with high demand variability across review periods and low case pack size. Our numerical analysis with real life data indicates potential cost savings that are considerable for the retailing industry with its tight margins and rigorous focus on cost-efficiency (a reduction of up to 66.27\% in optimality gaps). Large cost savings can be already achieved with a simple distinction between weekday and weekend sales, provided that the weekend is defined appropriately for each item (the optimality gap goes down to an average of 1.60\%). Doing so improves the return on investment in automated store ordering systems that are equipped with the capabilities of accounting for non-stationary demand.

As our model is not specific to a particular demand pattern, it may also be employed in ultra-high selling retail outlets with multiple deliveries per day, for instance. The smaller the review period, the more accurate the information for shelf replenishment. Moreover, albeit the focus of this research was the retailing industry, we believe the insights to be of interest to similar settings, where product values are low compared to handling costs, demand is non-stationary and non-availability is penalized by customers.

In order to limit the complexity of the inventory problem at hand, we excluded minimum shelf inventory requirements, shelf space restrictions (we assume shelf space to be sufficient for
demand during lead time), and perishability concerns in our analysis. Incorporating minimum stock requirements and shelf space restrictions into the model may provide interesting insights under non-stationary demand (see also Curşeu et al., 2011). Also, we investigate a single-item, single-location problem. Future research could work towards building multi-item and/or multi-echelon models and investigate workload scheduling and balancing in the retail supply chain (such as adjustments to delivery times as well as seasonal capacity constraints and cost) and at the store level. Lastly, we do not consider promotional events. Future research could work towards understanding promotional non-stationarity, e.g. the item-level intraday and across-days effects of various types of promotions. Combining the insights of promotional demand variation with our non-stationary demand inventory model accounting for ordering and handling costs may provide additional insights to retail marketers and logistics managers for increasing the return on promotions. Finally, another possible avenue for future research is the study of easy-to-implement heuristic policies which incorporate demand seasonality. For example, it would be interesting to study how well a \( (s_d, S_d, nq) \) policy, with \( d \) being the season index, would perform compared to the optimal policy used in this paper.

**Acknowledgments**

The authors would like to thank the retailer (who wishes to remain anonymous) who provided the data used in Section 6.

**References**


**Appendix**

Figure 11 shows the observed and (Poisson) expected frequency of cigarette sales (global brand, box of 10) during the lead time in 2010.
Figure 11: Frequency of cigarette sales during lead time in 2010